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LETTER TO THE EDITOR

Gauge invariance and the vortex glass

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Abstract. A model of disordered arrays of weakly coupled superconducting grains and a related random-gauge model are discussed. Evidence, from numerical studies of the zero temperature scaling behaviour of the stiffness, for a non-zero temperature spin-glass phase in the random-gauge model and in the strong magnetic field limit of the disordered arrays is presented. The universality class of a frustrated system may depend on whether the interactions have local gauge invariance.

One of the explanations of the unusual magnetic properties [1] of the high-T_c superconductors in the mixed state postulates [2] is that the magnetic flux lines in that state are frozen into a random array and not into a periodic Abrikosov lattice. In such a state, called the vortex glass, the spatial average of the square modulus of the Cooper-pair wavefunction is expected to be non-zero even though the average of the wavefunction itself is zero. Does such a state exist at non-zero temperatures in three dimensions? Resistivity data for films of $YBa_2Cu_3O_7$ seem to suggest that it does [3].

The possibility of glass-like behaviour in the mixed state of a superconductor arises because the phase of the complex order parameter changes discontinuously across structural defects. The Josephson effect, however, causes coupling between nearby regions ('grains') of uniform phase. When the magnetic field $H=0$, the coupling promotes phase coherence between grains. If the phase is represented by a two-component, or XY, spin, the coupling between spins representing nearby grains is ferromagnetic. In the presence of H, however, the coupling may not align the phases at two points because the line integral of the vector potential, A, between the points also contributes to the phase difference between them.

Using the standard expression for the interaction energy of a Josephson junction [4], this physics is captured by a random-grain model [5] described by the XY Hamiltonian

$$H=-\sum_{\langle ij\rangle}^{N} J_{ij}\cos(\phi_i-\phi_j-A_{ij}). \tag{1}$$

Here, $\langle ij\rangle$ denotes a summation over neighbouring grains located on a topologically disordered lattice and the gauge factors A_{ij} are given by

$$A_{ij}=2\pi/\Phi_0\int_i^j A\,\mathrm{d}l \tag{2}$$

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where Φ_0 is the flux quantum for a Cooper pair. The Josephson coupling constants J_{ij} are approximated by a single constant J, chosen to be 1 for simplicity. We shall choose the symmetric gauge, in which case

$$A_{ij} = \pi/2\Phi_0 H[(x_i - x_j)(x_i + x_j)]. \tag{3}$$

The question regarding the existence of the vortex glass now becomes: does the spin model defined by the Hamiltonian (l) have a finite-T spin-glass phase in $D=3$? Two recent studies of the model offer conflicting answers. First, Huse and Seung [6] carried out a Monte Carlo simulation of a simplified version of model (1) in which the gauge factors A_{ij} are taken to be random numbers satisfying $A_{ij} = -A_{ij}$ and distributed uniformly between 0 and 2π. The simulation suggested that the simplified model, which we shall henceforth call the random-gauge model, might have an Ising-spin-glass transition. Second, an ε-expansion about $D=6$ on a cubic field theory [7] for a gauge glass was interpreted [8] to imply a lower critical dimensionality greater than 3.

The Huse–Seung result is very interesting, because critical phenomena in spin systems usually depend on spin and space dimensionalities and the random-bond XY model—which may be recovered from the random-gauge model by choosing the A_{ij} to be randomly 0 or π—is known not to have a finite-T spin-glass phase in $D=3$ [9]. This makes it imperative to study for the random-gauge model quantities such as the scaling stiffness that turned out to be useful in unravelling the low-T phase of the Ising spin glass [9–12]. Such studies must be carried out directly in $D=3$, rather than in an ε or any other expansion about the mean-field limit, because the relevance of those expansions for the replica symmetric spin glass has been questioned recently [13].

In this letter we report results of numerical studies of the scaling stiffness of three models: the random-grain model, (1), for several values of H, the random-gauge and the random-bond models†. We find that the behaviour of the random-gauge model is similar to that of the random-grain model at high fields: In both cases, we find evidence for a spin-glass phase. But at intermediate fields, the random-grain model shows a complex behaviour.

The basic quantity of interest in the $T=0$ scaling theory is a scale-dependent coupling energy, $\delta E(L)$, which is determined by studying the sensitivity of ground-state energies of blocks of length L to changes in boundary conditions. We considered 3D blocks with periodic boundary conditions applied in two directions and either periodic or antiperiodic boundary conditions in the third direction. For each sample we determined the energy difference ΔE between ground-state energies with periodic and antiperiodic boundary conditions. We defined $\delta E(L)$ as $\langle|\Delta E|\rangle_c$, where $\langle\ldots\rangle_c$ is the average over samples. For unfrustrated systems δE coincides with the absolute value of the mean $\Delta = \langle\Delta E\rangle_c$, but for perfectly frustrated systems Γ should vanish since on average such systems do not prefer either periodic or antiperiodic boundary conditions.

If the ground state is frustrated but ordered, one expects

$$\delta E(L) \approx YL^{\theta} \qquad \Gamma \approx 0 \tag{4}$$

as L tends to infinity. For systems below the lower critical dimensionality, θ is negative and a phase transition occurs at $T=0$. On the other hand, for systems above the lower critical dimensionality θ is positive and there is a non-zero T_c.

† One of the earliest studies of the effect of impurities on the Abrikosov lattice used what may now be called a random-critical-temperature Landau-Ginzburg model. Although Larkin [14] argued in the context of that model that the Abrikosov lattice might be destroyed, studies of such models for random magnets suggest that they may not have a spin glass phase; see e.g. Sherrington [14].

We considered cubic samples with $L = 3, 4$ and 5. In the case of the random-grain model the grains were first arranged on a cubic lattice of lattice constant, $a = 1$, and then were displaced to random positions on surfaces of spheres of radii 0.2 centred on the nodes of the lattice. This is a 3D generalization of the planar disordered Josephson array studied by Morgenstern *et al* [15]. In our simulation, the magnetic field pointed along one of the lattice axes and its magnitude was measured in units of $2\Phi_0/\pi a^2$.

The ground-state energies were found using the following procedure. Starting with a random spin configuration the system was quenched to $T = 0$; a sequential optimal alignment of the spins was carried out until the total energy changed by less than 0.01%. A large number, K, of starting configurations were considered, with K chosen so that ΔE asymptoted to a constant value for a given L and H. We found that this procedure with a large K was more efficient in finding the ground-state energy than the slow Monte Carlo cooling or simulated annealing. We studied 100–200 samples except for the case of the $L = 5$ random-grain model with $H = 6$. The statistics in the case are based on 50 samples, because we had to take K as large as 2 [16].

Figure 1 shows the behaviour of δE as a function of L for the random-bond model, the random-gauge model, and the random-grain model for $H = 6$. The error bars were obtained by estimating the effects of finite statistics. In the case of the random-bond model δE decreases with increasing L with $\theta \approx -1$, in agreement with [9].

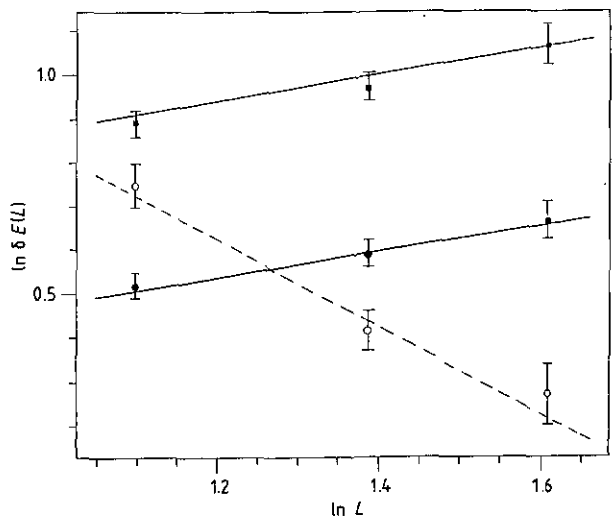

Figure 1. Dependence of $\delta E(L)$ on L for three models: The random-bond model (open circles), the random-gauge model (black squares), and the random-grain model for $h = 6.0$ (black circles). The full curves have a slope of 0.3 and the broken curve -1. All lines are guides to the eye.

The random-gauge model, by contrast, shows different behaviour: δE increases with increasing L, suggesting a non-zero T_c. Our data suggest θ to be of order 0.3, which exceeds the value of 0.19 obtained for Ising spin glasses [8, 9] using system sizes comparable to ours. The two models may thus be in different universality classes. It should be noted that the experiments [3] find $\nu = 1.7$ from measurements of the nonlinear I–V curves, compared with $\nu \approx 1.12$ found for Ising spin glasses [8].

We found that the behaviour of the random-grain model has a complicated dependence on the magnetic field. For weak fields H, there are even–odd effects reminiscent of antiferromagnets. The system seems to be completely unfrustrated for H less than 0.2: Γ has the same magnitude as δE (for L up to 6), so that the system is like a disordered ferromagnet or antiferromagnet [16]. The behaviour simplifies in the large field limit. It becomes possible to extract a value for the exponent θ with reasonable confidence at $H = 6$. This value, which stays the same for the highest fields studied, $H = 20$, is consistent with that found for the random-gauge model, as may be seen in figure 1.

Soon after the discovery of high-T_c superconductivity, the random-grain model was invoked to understand irreversibiliy effects in the magnetic measurements on those superconductors [15]. We have also studied such effects using a local mean-field theory for the magnetization and exploring the difference between field-cooled and zero-field-cooled values of the magnetization. We find, in agreement with earlier Monte Carlo simulations [17], that such effects set in for H on the order of 0.03, that is, at field values for which the model is not frustrated. This suggests that at onset, for fields just above the lower critical field, these effects may arise from some slow dynamical processes such as flux creep [18]. But the model must be explored more carefully, for small and intermediate fields, before anything definitive can be said on this subtle point.

Retrospectively, it is not surprising that the random-bond and random-gauge models have different behaviours because the models are fundamentally different even though both are systems of XY spins. Like thhe random-bond Ising spin glass the random-gauge model has a local gauge invariance†. This invariance means that the energy of any equilibrium configuration of the gauge model will not change if the spins are rotated by arbitrary amounts as long as the random gauge factors between nearest-neighbour spin pairs are also suitably adjusted. The random-bond model, by contrast, lacks this local invariance. The random-bond model only has an Ising-like gauge invariance. However, since the random-bond model and the Ising spin glass have different behaviour, it is clear that the symmetry of the local gauge invariance alone does not determine the universality class. The fact that the random-grain model does not order for a range of intermediate fields, for which the gauge invariance is not exact, further underscores the significance of the invariance for the vortex glass.

We conjecture that a general principle may underlie the different behaviours we obtain for the random-bond and random-gauge models. The principle may state, for example, that the critical dimensionalities of spin-glass models depend not only on the spin dimensionality but also on whether the interactions have local gauge invariance. It should not cause surprise, therefore, if a model even of Heisenberg spins had an ordered spin-glass phase in $D = 3$ if the competing interactions between the Heisenberg spins had the property of gauge invariance outlined above. A random-gauge model for Heisenberg spins can be written in terms of Euler angles. The study of such a model will be interesting for at least two reasons. First, what happens to the Higgs phase when non-Abelian gauge fields are not dynamical but are quenched‡? Second, such a model may be relevant to systems that have three-dimensional order parameters, such as nematic liquid crystals, when they are disordered.

The ultimate test of the relevance of the random gauge model to the vortex glass will be whether there is quantitative agreement between a measurable exponent and

† Random gauge models for disordered magnets were introduced by Hertz [19]. The random-gauge model may be a discrete lattice version of the U(1) invariant model discussed by him.

‡ The phase diagram of a theory of Higgs field coupled to dynamical gauge fields is discussed by Fradkin and Shenker [20].

its analytic estimation. Such analytic studies are underway and the results will be reported elsewhere.

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Note added. We have learned from Dr Tokuyasu that scaling studies similar to our own are being carried out on the random gauge model in $D=2$ and 3 by A P Young, M P A Fisher and T A Tokuyasu.

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